

History Of Mathematics: Impacts On Mathematics Learning And Teaching For The Development Of Critical Thinking

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Abstract:

Background: The history of Mathematics has often been neglected in Mathematics Education over the years, appearing, at best, associated with anecdotes or biographies that contribute little to the construction of mathematical knowledge. However, by exploring the History of Mathematics, human aspects such as controversies and conflicts between scientists are revealed, as well as debates that resulted in both theoretical advances and setbacks, stagnations, and modifications of concepts over time. This article aims to highlight the relevance of incorporating the history of mathematics into the educational environment, analyzing its contributions inside and outside the classroom, from the origins of mathematics to its application in teaching. Innovative approaches to teaching the history of mathematics are proposed, demonstrating its importance for student learning.

Materials and Methods: This theoretical study was conducted based on the issues discussed within the Study and Research Group in Mathematics Education (GEPEMA) at the Federal Institute of Education, Science, and Technology of Maranhão, using a bibliographic review and careful analysis of texts.

Results: This enabled us to present suggestions that allow students to learn Mathematics in a more critical way, understanding the historical stages involved in the creation of mathematical concepts and models.

Conclusion: We conclude that the history of Mathematics can be a powerful pedagogical tool, capable of promoting mathematical thinking and enriching the teaching-learning process.

Key Word: History of Mathematics. Mathematics. Learning. Teaching.

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I. Introduction

Throughout history, the processes of teaching and learning have been characterized by the continuous search for strategies in planning, mediation, and evaluation that promote students' appropriation of knowledge. More recently, there has been an emphasis on the development of critical thinking, understood as the ability to evaluate judgments based on logical reasoning. In this context, the development of critical thinking has been an essential historical practice, allowing individuals to validate their claims and question their own conclusions (Baroni, 1999).

Specifically, knowledge of the history of mathematics has played a crucial role in various philosophical, sociocultural, and epistemological discussions. In the epistemological field, the history of mathematics has enabled a critical analysis of historical trends concerning the nature of mathematics, its beliefs, and conceptions, with special attention to the relationships between mathematics and its teaching (Silva, 2017).

From a sociocultural perspective, the history of mathematics offers a reflection on the conditions and circumstances that motivated the development of certain mathematical knowledge, demonstrations, and proofs, highlighting the contexts that drove the advancement of this discipline. Moreover, the history of mathematics has fostered philosophical debates on the elements that promoted the construction of new concepts or, conversely, constituted obstacles to that construction (Lima, 2015).

The analysis of the history of mathematics encompasses multiple dimensions, with some authors suggesting a typology based on classifications of research products and historical analysis. Guacaneme (2010) identifies ten types of history of mathematics, where some refer to the object of study, such as biographies, original versions of mathematical works, translations, or the study of mathematical concepts and problems, further differentiating between original and secondary sources.

Other typologies consider the attention to sociocultural aspects and the development of mathematical works, as well as the depth of historical analysis. Regardless of the adopted typology, it is essential to define how the history of mathematics will be incorporated into teaching processes: as a specific content, as a pedagogical

tool, or as an element that contextualizes cultural studies on mathematics, including its public image and social role (Baroni, 1999).

The discussion on the interactions between the history of mathematics and mathematics education has gained prominence on the international scene. Despite numerous research projects, conferences, and publications on the topic in recent decades, there is still no clear consensus in the academic community regarding the contributions that history can offer to reflection in mathematics education, highlighting the importance of deepening the study of this issue.

This article seeks to emphasize the importance of integrating the history of mathematics into the teaching of this discipline, presenting the proposals and ideas of various authors and researchers who have dedicated themselves to the topic, as well as discussing the reasons and needs that have led to the revival of this approach, which is now strengthened by the formation of specialized working groups in the area. Thus, the objective of this work is to present an analysis of the relevance of the history of mathematics in the learning process.

II. Material And Methods

The methodology adopted for this study was based on a bibliographic and descriptive research approach, aiming to investigate the ideas of various authors and researchers who address the subject in question. After this investigation, we conducted a synthesis of these ideas, integrating and refining them according to the research objectives.

Fonseca (2002) emphasizes the importance of descriptive research, stating that its data should be collected in their natural environment, recorded for study, to allow the researcher to develop new perceptions about specific phenomena. The contact with articles published in scientific journals, dissertations in the field, and works by authors who discuss the central theme of this work provided the necessary foundation for a thorough academic reflection and for seeking answers to the raised questions.

Marconi and Lakatos (2007) state that the main purpose of research is to discover answers to proposed questions; it is based on a problem that needs to be solved, and the hypotheses raised may be confirmed or refuted. When mentioning descriptive research, we refer to the study of specific characteristics of a particular group, using appropriate instruments to verify what is being investigated (Gil, 2008). In the context of this work, the analysis will be directed toward the value of the history of mathematics as a didactic tool.

For the conduction of this study, the qualitative method was used. According to Minayo (2008, p. 57), the qualitative method is especially suitable for studies on history, representations, and beliefs, interpersonal relationships, perceptions, and opinions, that is, the products of interpretations that human beings make throughout their lives, shaping their material artifacts and themselves.

Minayo also argues that qualitative research is the most appropriate for investigating specific and focused sets of social issues, from the perspective of social subjects, and for analyzing discourses and documents. The qualitative approach includes the empirical part and progressively systematizes knowledge, seeking to understand the internal logic of the set. Furthermore, the author emphasizes that theory and methodology must go hand in hand, inseparably, where the methodology must have a clear, coherent, and well-developed instrument capable of guiding theoretical challenges into practice (Minayo, 2001). Thus, it is essential to establish solid methodological foundations for research.

Bibliographic research plays a crucial role in conceptualizing the terms to be addressed in the work, offering theoretical support based on books and articles that discuss the topic under consideration. According to Gil (2008), one of the main advantages of bibliographic research is allowing the researcher access to a significant volume of information, often greater than that obtained in descriptive research.

III. Result

The historical dimension of mathematics in its teaching is often justified by two main reasons: history provides an opportunity to broaden our understanding of what mathematics is and allows for a better assimilation of the involved concepts and theories (Barbin et al., 2000). In other words, it is expected that by studying the history of mathematics, both students and teachers will gain a deeper understanding of mathematical concepts, recognizing how they have developed over time and how this can change the way mathematics is perceived.

The same authors emphasize that these reasons are interconnected: the history of mathematics can, firstly, transform the teacher's perception and understanding of the discipline; secondly, this transformation directly impacts the way mathematics is taught, which ultimately affects how students perceive and understand mathematics.

By contextualizing and humanizing mathematics, teachers and students begin to see it not as an abstract entity but as a product of human activity, shaped over centuries to meet the needs of civilizations. Presenting the creators and their contributions, including the mistakes and imperfections they faced, humanizes the discipline, similar to what occurs in areas such as Literature. When the development of mathematical concepts is understood as a continuous process of reflection and improvement, mathematics is no longer seen as a fixed collection of

immutable truths but is recognized as a practical tool used by humanity to solve problems and challenges throughout history (Baroni, 1999).

Barbin et al. (2000) also state that the image of mathematics can be transformed by contrasting its formal presentation with a heuristic approach provided by history. The heuristic view, associated with constructivism, suggests that knowledge is gradually constructed, with concepts being clarified through the resolution of new problems. History, in this context, serves as a source of reflection for the teacher, who comes to understand the time it took for humanity to develop and accept certain knowledge, which can help adjust expectations regarding students' learning time for certain content. Thus, historical analysis can assist the teacher in identifying why certain concepts are "difficult" for students and in developing more effective teaching strategies.

Knowledge of the history of mathematics also enables the teacher to better understand the stages of the learning process and to propose problems inspired by historical contexts. For example, the historical difficulties in transitioning from numerical algebra to symbolic algebra can help teachers address the conceptual challenges their students may face. Moreover, students' responses to historical problems gain new meaning when compared to the solutions presented by mathematicians over time. Teachers can adopt a constructive attitude toward students' errors or focus on producing various answers to a specific problem, connecting it to what students already know (Baroni, 1999).

Guacaneme (2011) argues that the study of the history of mathematics requires and promotes both professional and personal skills, in addition to mathematical knowledge itself. Reading, writing, listening, finding sources, discussing, and analyzing mathematical topics develop sensitivity, tolerance, and respect for non-"conventional" forms of expression and problem-solving, as well as stimulate persistence and encouragement in the face of adversity. By recognizing the contributions of different cultures and ethnicities to mathematics, values such as tolerance and diversity are promoted, highlighting that knowledge is produced by all cultures. Teachers, therefore, can provide a richer cultural context for mathematics teaching by situating it in history.

According to Silva (2017), one of the reasons for using the history of mathematics in teaching is that if mathematical theories are presented only in their final formulation, without historical interpretations, students may develop a misguided understanding of mathematics, perceiving it as an artificial creation disconnected from practical work and real-life contexts. This view can be corrected when students, through historical facts, understand that mathematics has played a crucial role in all areas of human life since its origins. This allows them to acquire a deeper and more interesting understanding of mathematical concepts and to perceive mathematics not as an isolated discipline.

Integrating the history of mathematics into teaching helps students realize that mathematics is a constantly evolving system of knowledge, interconnected with other areas of science. Pedagogically, this helps students develop a scientific view of the world, recognizing the central role of mathematics in the cultural development of each era. By understanding that mathematical truths are the result of long and difficult processes of discovery, students come to accept that errors, doubts, intuitive reasoning, discussions, and alternative approaches are not only legitimate but also essential for the continuous formation of mathematics (Karaduman, 2010).

Studying the history of mathematics in the classroom clarifies why certain topics are taught, offering students a deeper understanding of the role of mathematics in solving everyday problems and in the evolution of human knowledge.

Throughout the centuries, mathematics has been fundamental to our daily lives, present in all areas and enabling remarkable achievements, such as space exploration. There are various ways to interpret mathematics, including artistic expressions such as theater and music. Mathematics is present in all of us, waiting for the right moment to manifest. Everyone has the capacity to understand and use mathematics. The history of mathematics, when used in teaching, should be presented in a "pedagogically oriented" way, as a living, human, enlightening, and dynamic narrative that inspires future generations (Miguel, 1997).

IV. Discussion

The importance of integrating history into teaching and learning processes, especially in mathematics, has been widely recognized. However, the question arises as to how this tool can be effectively applied in the educational environment. International trends and organizations have suggested a categorization of the levels of use of history, establishing three main approaches: chronological, logical, and pedagogical (Lopes & Ferreira, 2013).

The first approach, chronological, deals with the historical development of concepts, definitions, and other aspects of mathematics. An example would be the study of the evolution of the decimals of the number pi, from antiquity to the present day. Additionally, this level addresses the historical need to define terms such as the number zero, among others.

The second approach details how history contributes to the development of logical intuition in mathematics. By exposing the process through which scientists and mathematicians developed various theories,

students can understand the difficulties faced, the solutions found, and the mistakes made over time. This analysis allows students to demonstrate the nature of axiomatic systems and logical reasoning, as well as the mechanisms of proof. For the teacher, this is valuable as it can anticipate obstacles in the construction of mathematical knowledge and establish effective strategies to overcome them.

For example, this approach allows for the analysis of practical problems that originated the Fibonacci sequence, the review of graphical proofs of the Pythagorean theorem, or the discussion of difficulties in constructing irrational numbers.

Finally, the third approach considers the history of mathematics as an inexhaustible source of ideas and pedagogical strategies. This perspective allows teachers to approach the teaching of concepts, processes, and algorithms in light of the historical development of mathematics. An example would be a historical review of a specific topic, such as the biographies of mathematicians who contributed to the development of a theory, or the creation of a timeline showing the evolution of a concept. Studies indicate that the use of history in the mathematics classroom can revitalize the discipline, transforming it from a science perceived as static into a living area, full of historical development and practical applications. When students understand the meaning of the content, learning becomes more effective. Additionally, the teacher can adjust their approach, recognizing that topics that took a long time to develop historically also require adequate time to be assimilated by students. Knowing the history of mathematics allows the teacher to anticipate the difficulties that their students will face and prepare teaching strategies that reflect the historical development of the discipline (Barbin et al., 2000).

Moreover, the historical perspective suggests that mathematics should not be seen as a sequence of chapters in a book but as a dynamic intellectual activity, involving different ways of thinking about mathematical concepts and tools. Thinking of mathematics as a continuous activity, rather than a finished product, values problem-solving, conjecture, and intuition (Lopes & Ferreira, 2013). Santos and D'Ambrosio (2007) discovered that by comparing ancient mathematical problems with their modern solutions, both teachers and students can appreciate the importance of symbols and procedures used in current mathematics. Roque (2012) also points out that integrating history can reveal connections that would otherwise be imperceptible since mathematics often emerged to solve problems in seemingly unrelated disciplines.

However, teachers must be aware of the difficulties that may arise, such as the lack of time, resources, and adequate preparation. The use of the history of mathematics in classrooms can generate discussions within the mathematical community, especially among those who value the concrete results of mathematics (such as theories, theorems, and proofs) over the historical process that shaped them (Guacaneme, 2011).

It is suggested that the use of mathematical history be diversified, not limited to anecdotes or brief accounts. For example, when introducing content, one could analyze historical errors and encourage students to discuss and decide whether the proposed solutions were valid or not. Another approach would be to present ancient problems that had extraordinary solutions or that remain unsolved, problems whose resolution gave rise to entire theories or that became the focus of recreational studies.

The history of mathematics offers a rich source of problems that can be explored in a playful manner. Great moments and theoretical challenges that marked history can be transformed into recreational activities, where play and its pedagogical components occupy a central place (Santos & D'Ambrosio, 2007).

From this perspective, history can be used to promote a differentiated approach to mathematical knowledge. Historical data can be converted into playful activities. Problems related to infinity, paradoxes, formal systems, or non-Euclidean geometries have played important roles in the history of mathematics and constitute essential knowledge cores. Developing playful strategies to encourage an informal encounter with these concepts can be a great motivation in the learning process (Lima, 2015).

The possibilities of using history in teaching are varied and not only enrich the teaching of mathematics but also stimulate curiosity, creativity, interest, and the desire to learn in students. Additionally, they promote a change in students' perception of mathematics and in teachers' pedagogical approach. Finally, the recognition of the historical development of humanity, and particularly of a discipline like mathematics, allows people to adapt to the changes that society faces.

V. Conclusion

This attempt to characterize the various ways in which the History of Mathematics impacts Mathematics Education reveals a promising scenario. However, it is important to emphasize that this process is not simple, nor does it imply that all experiences are always successful. We fully recognize the complexity of the problem, which involves multiple dimensions, such as the type of knowledge addressed, teacher training, curriculum specifics, teaching strategies in relation to students' knowledge, conceptions of mathematics, history, and didactics, as well as the interests of students and society.

The complexity of this topic highlights the need to continue reflecting and advancing through educational proposals and practices, training programs, diversified dissemination strategies, and, of course, research projects. Initiating studies on the history of mathematics and mathematics education in each country is not just a theoretical

luxury but a fundamental cultural necessity to recover the history of ideas, sciences, and arts in our own context. This process also presents itself as an excellent vehicle for self-development, in addition to being able to engage and inspire other teachers in the study of the history of mathematics.

In the context of the History of Mathematics, the teacher finds a powerful means of self-formation, which allows for a deeper understanding of mathematics and the inherent difficulties in its transmission. This facilitates the path from teaching to learning, providing a valuable tool for the development of the capacity for pedagogical renewal and adaptation. Additionally, this approach enhances learning, transforming it into a constant rediscovery.

The History of Mathematics reveals itself as an inexhaustible source of didactic material, offering ideas, thought-provoking problems, and, to a great extent, entertainment and intellectual recreation. Furthermore, it provides personal, scientific, and professional enrichment, which can be utilized by teachers to motivate their work in knowledge transmission, while also contributing to the de-dramatization of mathematics teaching. Finally, the History of Mathematics, as a meeting point between the sciences and the humanities, serves as a powerful tool to culturally enrich mathematics teaching, integrating it harmoniously and interdisciplinarily into the academic curriculum.

Through the History of Mathematics, we can explore the questions that gave rise to different concepts, understand the intuitions and ideas that generated them, identify the origin of the terms, languages, and notations in which they were expressed, as well as the difficulties faced and the problems solved. This includes the context in which these ideas were applied, the methods and techniques developed, the way definitions, theorems, and proofs were forged, and the interaction between these elements to form theories. Additionally, history helps us understand the physical or social phenomena that these theories sought to explain, the spatial and temporal context in which they arose, how they evolved to their current state, and the connection of these themes with the cultural and everyday needs we face.

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